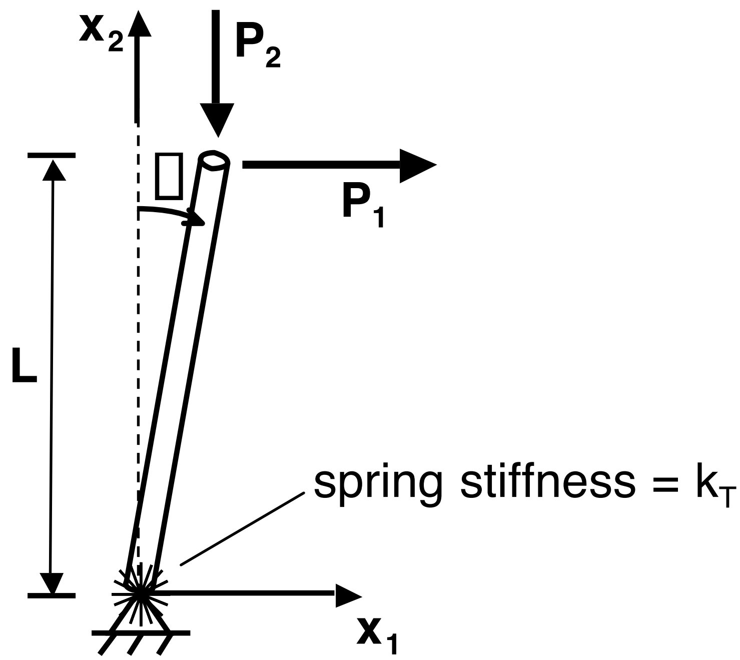
Unified Engineering Problem Set **Lectures: M13, M14, M15**
Week 10 Spring, 2008 **Units: M4.7**

M10.1 ***(15 points)*** Let's further explore the concept of instability via the configuration considered in lecture of the rod of length L pinned with a torsional spring of stiffness k_T and loads in each of the two directions. This is represented in the accompanying figure:



We extend the model in this case from the rod being infinitely rigid/stiff along its length to it being able to deform based on the basic rod model. This can be considered as being a linear spring with a stiffness constant proportional to the properties of the rod of the area (A), the modulus (E), and the length (L), resulting in the expression: $k_L = EA/L$. The length of the rod will thus change as it is loaded along its length. (NOTE that buckling of the rod as a column is not considered here).

(a) Determine the basic expression governing the rotational angle, θ, of the configuration as a function of the two applied loads and the parameters as defined.

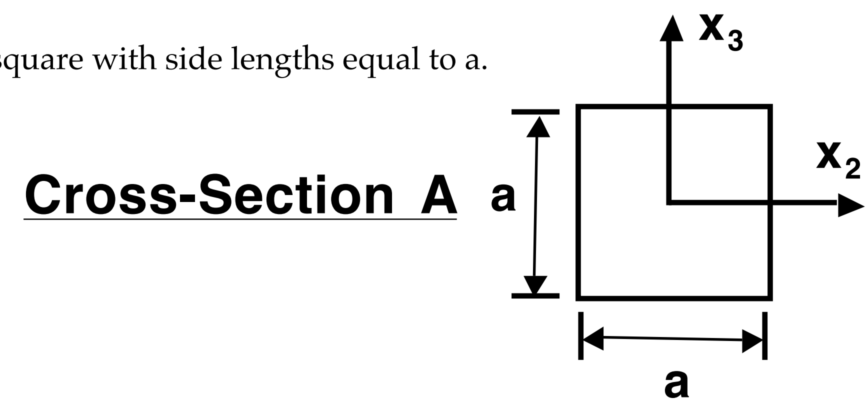
(b) Determine an expression for the critical value of the load applied along the length of the rod, $P_{2_{cr}}$, defined as where instability occurs.

(c) Plot this critical value of the load as a function of the value of the rod stiffness, EA, normalized by the ratio of the torsional spring stiffness to the rod length: $n = EA/(k_T/L)$. Explain the behavior that is observed.

M10.2 ***(15 points)*** A column of length L is clamped at one end and has no support at the other end. That end is loaded via a pressure of magnitude p and the overall load thus changes with and is proportional to the overall area of the cross-section. The column is made of a material with a modulus of E.

Different cross-sections are to be considered for the column. For each, determine the expressions to find the buckling load and the buckling mode.

(a) Cross-section A is square with side lengths equal to a.



(b) Cross-section B is circular with a diameter equal to a.